### Graphs of Trigonometric Functions Creation of a Mural

### Lesson 34-1 Periodic Functions

### Learning Targets:

- Identify periodic functions.
- Find the period, midline, and amplitude of periodic functions.

SUGGESTED LEARNING STRATEGIES: Close Reading, Paraphrasing, Create Representations, Vocabulary Organizer, Discussion Groups, Think-Pair-Share

An artist created this design to decorate a wall of the new transit center. The painters wondered if there is a mathematical description for the pattern to make it easier for them to reproduce it accurately.



The pattern repeats at regular intervals, or *periods*, so it is called a *periodic function*.

This graph shows a periodic function. You can extend it in both directions by repeating its shape.



This graph does not show a periodic function. Although it extends in both directions, you cannot predict its shape, because it does not repeat at regular intervals.



### Check Your Understanding

- **1. Attend to precision.** Sketch the graph of a periodic function. Explain why it is periodic.
- **2.** Sketch the graph of a function that is not periodic. Explain why it is not periodic.

2015 College Board. All rights reserved.

### Common Core State Standards for Activity 34

HSF-IF.C.7Graph functions expressed symbolically and show key features of the graph, by hand in<br/>simple cases and using technology for more complicated cases.  $\star$ HSF-IF.C.7eGraph exponential and logarithmic functions, showing intercepts and end behavior, and<br/>trigonometric functions, showing period, midline, and amplitude.HSF-BF.B.3Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for<br/>specific values of k (both positive and negative); find the value of k given the graphs.<br/>Experiment with cases and illustrate an explanation of the effects on the graph using<br/>technology. Include recognizing even and odd functions from their graphs and algebraic<br/>expressions for them.

# My Notes Image: Constraint of the second second

MATH TERMS

A **periodic function** is a function that repeats its values in regular intervals called periods.



# ACTIVITY 34

Guided

### **Activity Standards Focus**

In Activity 34, students analyze, graph, and write equations for parent trigonometric functions and their transformations, including phase shifts. They identify period, midline, amplitude, and asymptotes, as applicable.

### Lesson 34-1

### PLAN

Pacing: 1 class period

**Chunking the Lesson** Check Your Understanding #3 #4–6 Check Your Understanding Lesson Practice

### TEACH

### Bell-Ringer Activity

Find sin  $\theta$  and cos  $\theta$ .

**1.**  $\theta = 90^{\circ}$  [sin  $90^{\circ} = 1$ , cos  $90^{\circ} = 0$ ] **2.**  $\theta = 2\pi$  [sin  $2\pi = 0$ , cos  $2\pi = 1$ ]

**3.**  $\theta = 180^{\circ}$  [sin  $180^{\circ} = 0$ , cos  $180^{\circ} = -1$ ]

### Introduction Close Reading, Discussion Groups, Look for a

**Pattern** Students are asked to distinguish between graphs of periodic functions and functions that are not periodic. As students examine the graph of the triangle wave, ask guiding questions to help them find patterns. For example: How frequently does the graph cross the *x*-axis? [*every 4 units*] What is the interval at which the graph repeats itself? [*every 8 units*] Remind students to observe the graph's behavior with respect to *y*-values, as well.

### **Differentiating Instruction**

Some students, especially logical learners, may benefit from making a table of ordered pairs for the points shown on the graph of the triangle wave in order to identify numerical patterns.

### **Check Your Understanding**

Debrief students' answers to these items to ensure that they understand concepts related to periodic functions. For Item 1, ask students to identify the interval at which the graph repeats.

### Answers

- **1.** Graphs will vary but must repeat at regular intervals. Explanation: it repeats at regular intervals.
- **2.** Graphs will vary but must not repeat at regular intervals. Sample explanation: It does not repeat at regular intervals.

## ACTIVITY 34 Continued

### **Paragraphs Developing Math** Language

The new vocabulary terms for this lesson are *periodic function*, *period*, amplitude, and midline. As you guide students through their learning of these new essential mathematical terms, explain meanings in ways that are accessible for your students. When possible, provide concrete examples to help students gain understanding. Encourage students to make notes about new terms and their understanding of what they mean and how to use them to describe precise mathematical concepts and processes.

### **ELL Support**

The word *oscillate* is often used to describe the behavior of a periodic function. Definitions of oscillate often emphasize moving back and forth between two points, usually in a rhythmic motion. If students have trouble grasping the meaning, they may recognize a visual image or video of an oscillating wave on an oscilloscope. Many students may have seen one of these instruments in real life or on TV, although they may not have known what it is called.

### **3 Think-Pair-Share, Create Representations, Look for a Pattern, Visualization, Vocabulary Organizer**

If students are struggling to draw a rectangle around one repetition of the graph, have them look at the sample rectangle outlining the portion of the graph in the figure at the top of the page. Make sure they are identifying the correct boundaries of that rectangle by having them identify the coordinates of its four corners [(0, 2), (0, 6), (2, 2), and (2, 6)]. Some students may benefit from visualizing the rectangle as a tile-if copies of the rectangle are tiled end to end, the graph of the entire function is created. Also, note that at this point in the activity, many vocabulary terms are presented. Students should add these terms to their vocabulary organizer. You may wish to lead a class discussion about the terms as they are added to the classroom Word Wall.

### **ACTIVITY 34** continued To describe the design shown for the transit v feet center wall more precisely, you can define MATH TERMS its period, amplitude, and midline. Study the graph shown here. A period is the horizontal distance required for the graph of a periodic The portion of the design outlined by the function to complete one rectangle shows one repetition, or period, of repetition, or cycle. the function. The horizontal distance of one repetition is 2 units, so the period of the The amplitude of a function is half function is 2. the difference between the minimum and maximum values of the range. $2 \le y \le 6$ . Half of that distance between these two values is called the The **midline** is a horizontal axis *amplitude* of the function. Since $|6-2| \div 2 = 2$ , the amplitude of the that is used as the reference line function is 2. about which the graph of a periodic function oscillates. The horizontal line that runs midway between the maximum and minimum the line y = 4 is the midline of the function. 3. Look at the following graph. **a.** Draw a rectangle around exactly one repetition of the graph of the $-4 \le x \le 4$ or $0 \le x \le 8$ ; rectangles must include $-4 \le y \le 2$ . b. How wide is the rectangle? 8 units

- c. How high is the rectangle? 6 units How can you use the height to find the amplitude? Find one-half of the height,  $6 \div 2 = 3$
- d. Draw the midline of the function. Check students' work. What is the equation of the midline? y = -1

# Lesson 34-1

**Periodic Functions** 



The graph oscillates between y = 2 and y = 6, so the range of the function is

values of the function is the *midline*. Because 4 is midway between 6 and 2,

- function. Check student's work; rectangles can include, for example,
- What feature of the periodic function is the width of one repetition? the period